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Heavy Baryon Masses in Large N_c HQET

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Abstract

We argue that in the large N_c HQET, the masses of the s-wave low-spin heavy baryons equal to the heavy quark mass plus proton mass approximately. To the subleading order, the heavy baryon mass $1/N_c$ expansion not only has the same form, but also has the same coefficients as that of the light baryon. Based on this, numerical analysis is made.

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Heavy baryons provide us testing ground for the Standard Model. Those containing a single heavy quark, like Λ_c, Λ_b, $\Sigma_c^{(*)}$ and $\Sigma_b^{(*)}$, can be studied within the heavy quark effective theory (HQET) [1]. For complete calculations for them, some additional nonperturbative methods have to be used. In this Letter, we discuss the simple incorporation of large $\mathrm{N_c}$ [2] method in HQET.

HQET is an effective field theory of QCD in the heavy quark limit [1]. In a systematic manner, it fits the description for the heavy hadrons. Under the heavy quark limit, there is no heavy quark pair production. The large mass of the heavy quark which interacts with the light quark system with typical energy Λ_{QCD}, plays no role except for the total energy of the hadron. With the velocity super-selection rule, the heavy quark mass m_Q, which is defined perturbatively as the pole mass, can be removed by the field redefinition. The heavy quark field h_v is defined by

$$P_+ Q(x) = \exp(-i m_Q v \cdot x) h_v(x) \ , \tag{1}$$

where $P_+ = \frac{1}{2}(1+ \not{v})$. To the leading order of $1/m_Q$, the effective Lagrangian for the heavy quark is

$$\mathcal{L}_{\mathrm{eff}} = \bar{h}_v i v \cdot D h_v \ . \tag{2}$$

Besides the heavy quark symmetry [1], we note explicitly from Eq. (2) that the heavy quark becomes effectively massless (modula m_Q). The heavy hadron mass M is expanded as

$$M = m_Q + \bar{\Lambda} \ , \tag{3}$$

where $\bar{\Lambda}$ is the heavy hadron mass in the HQET, which is independent of the heavy quark flavors. The quantity $\bar{\Lambda}$ cannot be determined from the HQET further. It is at this stage, we apply the large $\mathrm{N_c}$ method.

As one of the most important and interesting method of nonperturbative QCD, large $\mathrm{N_c}$ limit [2] is often applied in spite of the realistic $\mathrm{N_c} = 3$. Nonperturbative properties of mesons can be observed from the analysis of the planar diagram, and

baryons from the Hartree-Fock picture. Recently, there are renewed interests in the large $\mathrm{N_c}$ application to baryons due to the work of Ref. [3] which shows that there is a contracted $\mathrm{SU}(2f)$ light quark spin-flavor symmetry in the baryon sector, by combining the large $\mathrm{N_c}$ counting rules and the chiral Lagrangian. Actually this symmetry can be directly derived in the Hartree-Fock picture [4], or by other method [5]. Similar result was also obtained before [6]. Further applications of this spin-flavor symmetry to heavy baryons are made by Jenkins [3] in discussing the baryon-pion couplings and the baryon hyperfine splittings. Interesting relations among the baryonic Isgur-Wise functions are obtained in Refs. [7] as well as [8]. Masses of the heavy baryons with any finite number of heavy quarks are studied by $1/\mathrm{N_c}$ expansion of QCD in Ref. [9].

Inspired by these approaches, we consider the HQET at the large $\mathrm{N_c}$ limit. Physically, the heavy quark limit and the large $\mathrm{N_c}$ limit are non-commutative. Different order of the limits corresponds to different picture. In the large $\mathrm{N_c}$ HQET, there is nothing new in the meson case. So we discuss the heavy baryons.

We argue that the mass of the s-wave low-spin heavy baryons in the HQET $\bar{\Lambda}$ equals to the proton mass in the large $\mathrm{N_c}$ limit. Let us continue thinking of the Hartree-Fock picture not in the full QCD, but in the HQET. The heavy baryons contain $(\mathrm{N_c}-1)$ light quarks, and one "massless" heavy quark. The mass or the energy of the baryon is determined by the summation of the energies of individual quarks. The kinetic energy of the heavy quark is typically Λ_{QCD} like that of the light quark. The interaction energy between the heavy quark and any of the light quarks is typically $\Lambda_{\mathrm{QCD}}/\mathrm{N_c}$. So the interaction energy between the heavy quark and the whole light quark system scales as Λ_{QCD}. However, the total interaction energy of the light quark system itself scales as $\mathrm{N_c}\Lambda_{\mathrm{QCD}}$. In the limit $\mathrm{N_c}\to\infty$, the light quarks drown the heavy quark. The energy of the heavy baryon is determined by its light quark system. This light quark system also dominates the proton in the large $\mathrm{N_c}$ limit. Therefore we come to the conclusion: in the large $\mathrm{N_c}$ limit, the masses of the s-wave low-spin heavy baryons

defined in HQET equal to the proton mass.

From the same logic as in last paragraph, we can easily deduce the results for the baryon-pion coupling constants. These constants are also determined by the light quark system. So they are the same for the light baryons and the heavy baryons. And the heavy baryon also has the light quark spin-flavor symmetry. These results are obtained by Jenkins in Ref. [3].

Of course, all the results are subject to $1/\mathrm{N_c}$ corrections which deserve more detailed considerations. The correction violates the light quark spin-flavor symmetry. Let us first discuss the spin symmetry violation in $\bar{\Lambda}$. The baryon mass can be written as

$$\bar{\Lambda} = N_c \Lambda_{\mathrm{QCD}} + c_1 J_l^2 / N_c \ , \tag{4}$$

where J_l is the angular momentum of the light quark system. The mass parameter c_1 is yet undetermined which is of order Λ_{QCD}. The factor $\mathrm{N_c}$ should appear so as to keep the $\mathrm{N_c}$ scaling for $\bar{\Lambda}$. In the extreme case while all the quark spins align in the same direction, $J_l^2 \sim {\mathrm{N_c}}^2$. Only by dividing a factor $\mathrm{N_c}$, has the term $\sim J_l^2$ in Eq. (4) the right $\mathrm{N_c}$ scaling. Note this term is $1/\mathrm{N_c^2}$ suppressed compared to $\mathrm{N_c}\Lambda_{\mathrm{QCD}}$. On the other hand, the light baryon mass m has the same form of $1/\mathrm{N_c}$ expansion,

$$m = \mathrm{N_c} \Lambda_{\mathrm{QCD}} + \tilde{c}_1 J^2 / \mathrm{N_c} \ , \tag{5}$$

where J is the baryon spin. Further, we argue in the following that

$$c_1 = \tilde{c}_1 \ . \tag{6}$$

Consider still the above extreme case, where in the mass $1/\mathrm{N_c}$ expansion, the subleading term becomes a leading one, $J^2 = \frac{\mathrm{N_c}}{2}(\frac{\mathrm{N_c}}{2}+1)$ and $J_l^2 = \frac{{\mathrm{N_c}}^2-1}{4}$. Because of the light quark dominance, we have $m = \bar{\Lambda}$ in the limit $\mathrm{N_c} \to \infty$. This immediately results in the conclusion given by Eq. (6).

Another lowest order $1/\mathrm{N_c}$ effect lies in the light quark flavor symmetry breaking. At the moment, we forget the spin symmetry violation. After including the baryons with strangeness number -1, the masses for the heavy and light baryons can be expanded as

$$\begin{aligned} \bar{\Lambda} &= \mathrm{N_c}\Lambda_{\mathrm{QCD}} + c_2(-S) \ , \\ m &= \mathrm{N_c}\Lambda_{\mathrm{QCD}} + \tilde{c_2}(-S) \ , \end{aligned} \tag{7}$$

respectively. Where S is the baryon strangeness number which can be 0 or -1. Again we will argue

$$c_2 = \tilde{c_2} \ . \tag{8}$$

In the expression (7), the spin symmetry is not violated. The strange quark spin decouples from the strong interaction. The only contribution of the strange quark mass to baryon masses is the strange quark mass itself. Therefore c_2 and $\tilde{c_2}$ are nothing but the strange quark mass defined in the large $\mathrm{N_c}$ limit. To the order $1/\mathrm{N_c}$, terms like I^2 and $I \cdot J_{(l)}$ should be included in the expansion (7). However, in the realistic case, $I = J$. These terms can be effectively absorbed into the term J^2 in Eq. (4).

For a complete analysis of the heavy baryon masses, $1/m_Q$ corrections have to be considered. To the order of $1/m_Q$, heavy baryon mass M is expanded as

$$M = m_Q + \bar{\Lambda} - \frac{\lambda_1}{2m_Q} + \frac{2\lambda_2}{m_Q}(S_Q \cdot J_l) \ , \tag{9}$$

where S_Q is the heavy quark spin and

$$\begin{aligned} \lambda_1 &= < H(v)|\bar{h}_v(iD)^2 h_v|H(v) > \ , \\ 2\lambda_2(S_Q \cdot J_l) &= -\frac{1}{4}Z_Q < H(v)|\bar{h}_v g\sigma \cdot G h_v|H(v) > \ , \end{aligned} \tag{10}$$

with Z_Q being the renormalization factor. In the leading order $1/\mathrm{N_c}$, λ_1 scales as unity and is independent of the light quark structure; λ_2 is vanishing. These can be seen directly from the definition (10) with light quark spin-flavor symmetry, and from the fact that λ_2 is zero for Λ_Q baryon. Therefore we arrive the following $1/\mathrm{N_c}$ expansion for λ_1 and λ_2,

$$\begin{aligned} \lambda_1 &= c_0' + c_1' J_l^2/{\mathrm{N_c}}^2 + c_2' S/N_c \ , \\ \lambda_2 &= c''(S_Q \cdot J_l)/\mathrm{N_c} + c_2'' S/\mathrm{N_c} \ . \end{aligned} \tag{11}$$

We perform the numerical analysis for the non-strange baryons in the following. The heavy baryon mass is presented in Eq. (9). For $\bar{\Lambda}$ and m, the $1/\mathrm{N_c}$ expansions are given in Eqs. (4) and (5) with $c_1 = \tilde{c}_1$. And for λ_1 and λ_2 in Eq. (11) with $S = 0$. To be consistent, the accuracy of the analysis is maitained to the order of $\frac{\Lambda^2_{\mathrm{QCD}}}{m_Q \mathrm{N_c}}$ and $\frac{\Lambda_{\mathrm{QCD}}}{\mathrm{N}^2_{\mathrm{c}}}$. That means the term c'_1 in Eq. (11) is also neglected. Formally the uncertainty will be due to $1/m_Q^2$ and $1/N_c^3$ corrections which are about 10 MeV. With the measured masses of proton, neutron and Δ, we obtain $\mathrm{N_c}\Lambda_{\mathrm{QCD}} = 866$ MeV and $c_1 = 293$ MeV. This gives $\bar{\Lambda}_{\Lambda_Q} = 866$ MeV and $\bar{\Lambda}_{\Sigma^{(*)}_Q} = 1060$ MeV. Although there is no data for c'_0, the following quantity can be predicted with the theoretical accuracy of 10 MeV,

$$
\begin{aligned}
\tfrac{1}{3}(M_{\Sigma_c} + 2M_{\Sigma^*_c}) &= M_{\Lambda_c} + \bar{\Lambda}_{\Sigma^{(*)}_c} - \bar{\Lambda}_{\Lambda_c} \\
&= 2479 \text{ MeV} .
\end{aligned}
\tag{12}
$$

Similarly the corresponding quantity for bottom quark is predicted as

$$
\frac{1}{3}(M_{\Sigma_b} + 2M_{\Sigma^*_b}) = 5835 \pm 50 \text{ MeV} . \tag{13}
$$

Eq. (12) shows that the recent proposed $\Sigma^{(*)}_c$ masses in a new interpretion [10] of heavy baryon spectrum are in 100 MeV deviation from our result. It also implies that $M_{\Sigma^*_c} = 2492$ MeV by taking $M_{\Sigma_c} = 2453$ MeV. Our numerical analysis actually is the same as that in Ref. [9].

Comparing with Ref. [9], what are the different points of this paper? We began with the HQET which gives a clear physical picture for heavy baryons, and emphasized the heavy baryon mass in HQET $\bar{\Lambda}$ is at the order of proton mass. Then we showed that the next to leading order $1/\mathrm{N_c}$ expansions of $\bar{\Lambda}$ and the light baryon mass not only have the same form, but also have the same coefficients. These points cannot be taken for granted in large $\mathrm{N_c}$ HQET. They justifies some of the numerical analysis of Ref. [9].

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